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## CAMERAS FOR PHOTOGRAPHING MISSILES IN HYPERSONIC FLIGHT

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## CAMERAS FOR PHOTOGRAPHING MISSILES IN HYPERSONIC FLIGHT

M. Paques and M. Lecomte<sup>1</sup>

## ABSTRACT

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The Institut Franco-Allemand at Saint-Louis (France) launches 11-mm diameter models at hypersonic speeds, using for observation in flight:

- (1) spark cameras for shadow setting;
- (2) a camera with speed compensated by a rotating mirror, at better than 1 percent, for shadow setting and emitted light;
- (3) an experimental electronic shutter camera for emitted light.

Finally, densitometric measurements enable processing of films.

It is shown that diffraction and depth of focus set a minimum size to the spread function of an image point, whatever the adjustment or the type of camera may be, and the main relationships permitting evaluation of the actual spread function for a given task are described.

*Author*

The photo equipment used at the Saint-Louis Institute is conceived to /38\* permit observation of a missile having a diameter ranging up to 11 mm, moving at speeds up to 8,000 meters per second, and launched by a light-gas cannon, in a tunnel approximately 300 mm in diameter, where a given atmosphere (pressure and content) can be established.

Three types of photographic equipment are used:

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\*Numbers given in the margin indicate the pagination in the original foreign text.

<sup>1</sup>Engineers and research scientists at the Institut Franco-Allemand de Recherches de Saint-Louis (France).

(1) Spark cameras where the missile picture formed by its shadow in the light beam produces a spark. These cameras are used to determine the exact position of the missile, and particularly to measure its speed (fig. 1).

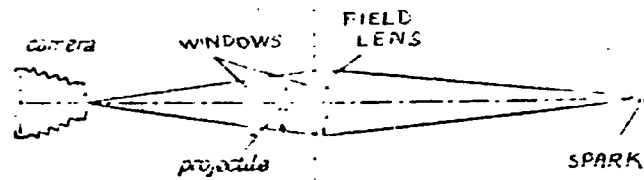


Figure 1

(2) A camera equipped with a rotating mirror, where the missile movement is compensated by a rotating mirror inside the camera, which causes an immobilization of the missile picture on the film. Figure 3 shows a simplified sketch of the camera.

This is in fact a complete camera, i.e., a picture is obtained whatever the position of the rotating mirror may be, when the phenomenon to be photographed is within the field of the camera. The reason for this is that it is impossible to synchronize the mirror and the phenomenon. In order to avoid a masking of the film by the concave mirrors, it was necessary to slightly 39 shift the mirrors sideways, thus bringing about slight maladjustments in relation to the sketch shown in figure 3. As a consequence, it is impossible to obtain a correct image on both films. The camera has been set in such a way so as to obtain a satisfactory picture on the upper film. When the image appears on the lower film, it will have such a distortion that a rectangle shall be seen as a parallelogram and shall give qualitative results only. The experiment has shown that sufficiently good pictures are thus obtained. Also, the possibility of not obtaining a picture because of the non-photographical equipment should not be ignored.



Figure 2. Picture taken with the rotating mirror camera.

An exterior optical system enables us to obtain for each shooting a shadow picture and an emitted light picture of the missile, separated by a time interval equal to that required by the missile to cover about 10 cm. An intermediary optical apparatus, enlarging 1:1, forms at the center of the camera field the picture of an axis point located about 10 cm forward. A light source is focused with a field lens so that the final picture of that source will be formed on the camera lens. A diaphragm is set up in the plane of the first

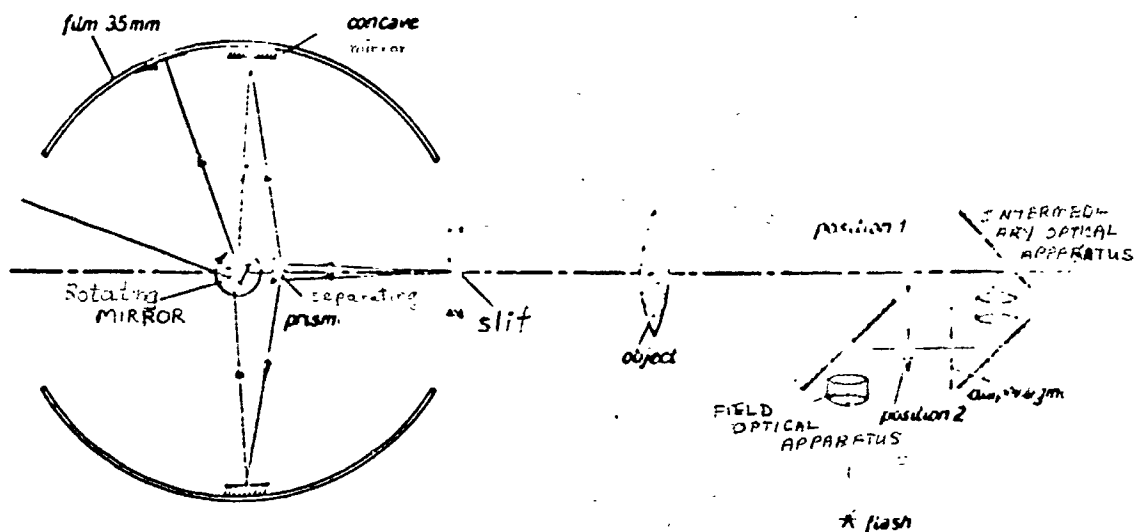


Figure 3

image of that source the light emitted toward the camera by the missile in position 2. This setting could be extended to a greater number of pictures for emitted light, but it would increase the losses due to reflection and difficulties of adjustment.

(3) Finally, an experimental electronic camera is used. It is equipped with an RCA 4449 tube and is employed exclusively with emitted light, as a comparison with the preceding camera.

These three types of equipment produce varied information usually involving errors, either systematic or contingent, which complicate the use of films obtained.

A list of information which one could obtain from a picture can be established:

- Velocity or, what is equivalent, the position at a given moment.
- Missile incidence.
- Shape and position of the shock wave.

- Removal effects.
- Trail.

Except in the latter case, we consider that we are facing a stationary phenomenon.

The picture deterioration causes having a noticeable effect are the following:

- the missile translation motion;
- the setting error resulting from the missile's transversal dimensions;
- the setting error due to the gaps between the path and the tunnel axis;
- diffraction;
- the film's responsive curve in density;
- the film resolving power limitation for various reasons, particularly the grain.

#### Error Resulting from the Missile Motion

This concerns the shifting of the missile image on the film during photography.

If we designate by  $h(t)$  the shutting function or the spark emitting curve, the picture of an object point which moves at speed  $v$  on the film will give a segment of a straight line, where its intensity will vary as  $h \frac{(x)}{(v)}$ . The picture will therefore be blurred and will be deduced from the clear picture by convolution with  $h \frac{(x)}{(v)}$ .

In the case of the spark camera, the light curve has the configuration shown in figure 4a, i.e., resulting in a relatively significant blur. Its rapid rise, however, makes it possible to spot with sufficient accuracy the missile position

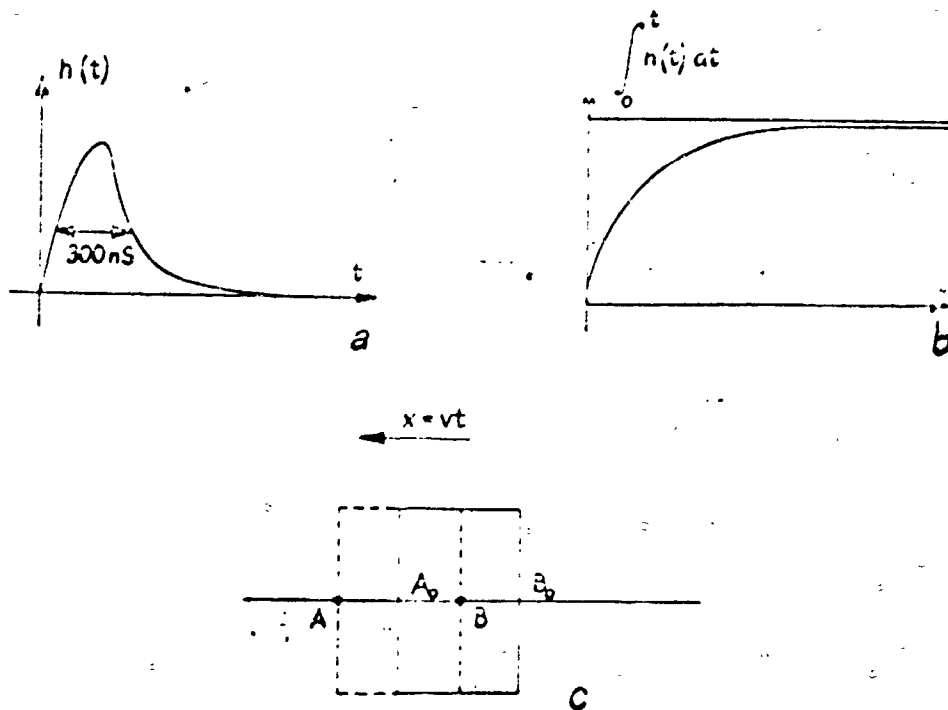


Figure 4

at the time of the spark. Indeed, let  $A_0 B_0$  be that position (fig. 4c) and  $A B$  its position at the end of a time  $t = \frac{AA_0}{v} = \frac{BB_0}{v}$ .

A point such that  $A$  will be hidden at the end of a time  $t$ , and will have received an illumination proportionate to

$$I_A = \int_0^t h(t) dt$$

A point such that  $B$ , hidden at the beginning, will be illuminated from a time  $t$ , which gives altogether

$$I_B = \int_t^\infty h(t) dt = \int_0^\infty h(t) dt - I_A$$

$\int_0^t h(t) dt$  has the configuration shown in 4b.

The swift beginning of the use is found again on the densitograms (fig. 5) enabling <sup>ms</sup> to plot the missile at the time of the spark, with relation to a marking which appears also on the densitogram. Using the known distance between two stations, the speed can be determined with sufficient accuracy.

In the case of the shutter camera, it is possible to eliminate, in principle, the blurring. In fact, since it is necessary to set up the speed a priori, it is only after the shooting that the missile's true speed is compared to its planned speed. There subsists a mistake having a 1 percent scattering.

If  $d$  is the width of the aperture,  $v_0$  the design speed and  $v$  the effective speed in the image space, we shall have the following values

- exposure time:  $\tau = \frac{d}{v}$

- picture length from a point (blurred)

$$\delta_{i,v} = \frac{d |v - v_0|}{v} = d\alpha$$

- length distortion coefficient on the missile

$$\frac{v_0}{v}$$

Typical values are

$$d = 5 \text{ mm}$$

$$v = 4.5 \text{ mm}/\mu\text{S}$$



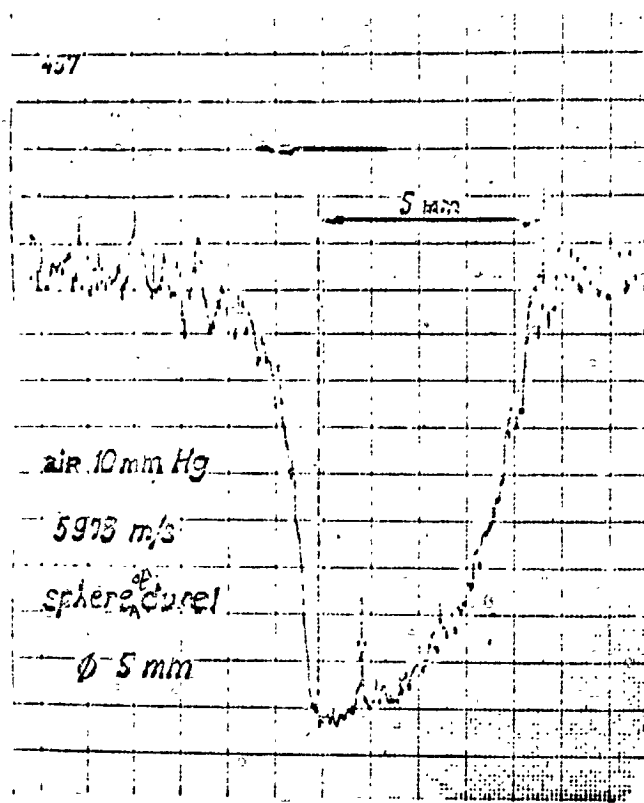


Figure 5.

$$\left| \frac{v - v_0}{v} \right| = \frac{1}{100}$$

Hence an exposure time in the order of:

$$\tau = 1.1 \mu\text{S}$$

and a blur of  $\delta_{iv} = 0.05 \text{ mm}$  (on the film).

With the electronic camera, unlike the rotating mirror camera case, the blurring effect is directly proportionate to the exposure time and the speed.

With the same notations as above, we obtain

$$\delta_{iv} = v \cdot \tau$$

or, brought back to the space object, with V speed of the missile

$$\delta_{ov} = V \cdot \tau$$

i.e., numerically with:  $\tau = 20 \mu s$

$$V = 8000 \text{ m/s.}$$

$$\delta_{ov} = 0.16 \text{ mm.}$$

Another expression of  $\delta_{iv}$  will prove useful subsequently; it involves G and the numerical aperture  $\theta$ . For a phenomenon of a given luminance, on a specific film, we must have as a first approximation the relationship

$$\frac{\tau}{\theta^2(G+1)^2} = \frac{I}{K}$$

$\tau$  = exposure time

G = enlargement.

K being proportionate to the luminescence and to ASA sensitivity.

If V designates the velocity and  $\delta_{ov}$  the blur of the space object, we have

$$\delta_{ov} = V\tau = \frac{V\theta^2(G+1)^2}{K}$$

and

$$\delta_{iv} = G \cdot V \cdot \frac{\theta^2}{K} (G+1)^2.$$

## Setting Errors

If a target gives of a P point a P' image (fig. 6), the image of an M point located ahead of P will form in the P' plane a homothetic spot of the lens diaphragm in the relation

$$k = \frac{M'P'}{M'O} = \frac{M'O - P'O}{M'O} = \frac{\frac{I}{P'O} - \frac{I}{M'O}}{\frac{I}{P'O}}$$

Let us have

$f$  = focal distance

$G_0$  = enlargement in P

$G$  = enlargement in M.

We then obtain

$$k = \frac{MP}{f} \cdot \frac{G \cdot G_0}{G+1} \approx \frac{MP}{f} \cdot \frac{G_0^2}{G_0+1} \quad (1)$$

where MP represents the setting gap in the space object.

If the relative numerical aperture  $\theta$  of the target is introduced, the distance of the spread function edges resulting from the blurring effect is deduced from (1),

$$\text{in the space image} \quad \delta_{if} = \frac{MP}{\theta} \cdot \frac{G^2}{G+1}$$

$$\text{in the space object} \quad \delta_{of} = \frac{MP}{\theta} \cdot \frac{G}{G+1}$$

With the spark camera, the edges parallel to the path, being in principle in the focus plane, must give a clear picture. In the case of perpendicular edges, the relationship (1) gives

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$$k = \frac{5.5}{450} \cdot \frac{(0.5)^2}{1.5} \approx 0.002$$

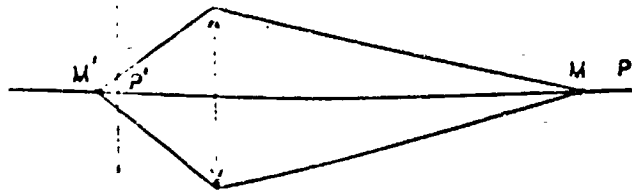


Figure 6.

The diaphragm being a 4 mm aperture in diameter, the corresponding spot is therefore 0.008 mm, i.e., extremely weak in front of the blur caused by the motion.

In the case of the rotating mirror camera, the lens diaphragm is more or less equivalent, in emitted light, to a rectangle having the dimensions

$$75 \cdot 12.5 \text{ mm,}$$

the long side of which is perpendicular to the path (fig. 7). In shadow setting, it looks more or less like a semi-circle having a 10 mm diameter arranged in the same manner as in figure 7. In the case of a cylindric missile crossing the tunnel axis, the distance from the setting central plane of the front side edge is that of the value of a radius. The image point of this edge (M, fig. 7) shall therefore be a rectangle, the small side of which determines the blur.

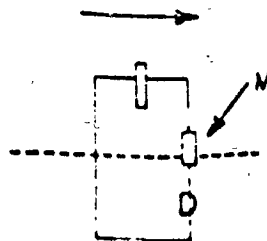


Figure 7.

Numerically,

$$\theta = \frac{700}{12.5} = 56 \text{ for emitted light}$$

$$\theta = \frac{700}{10} = 70 \text{ for shadow setting}$$

$$G \simeq G_0 = 0.6$$

$$MP = 5.5 \text{ mm}$$

$$\begin{aligned} \delta_{if} &= 0.022 \text{ mm for emitted light} \\ &= 0.017 \text{ mm for shadow setting.} \end{aligned}$$

If the front face is seen exactly through the edge, at the shooting and sight axis intersection, the blurring profile will be more or less represented by figure 8.

If the missile is not properly centered and is incident, the blur will generally increase. It can be reduced if the incidence is such that one sees only the front face edge and that the gap in relation to the shooting axis brings this edge towards the tunnel axis.

The electronic camera uses greater apertures, because the desired exposure times are very short and the tube gain (5-10) is not sufficient to compensate for the resulting light loss.

The experimental camera used a lens  $f = 200 \text{ mm}$ ;  $1/1.9$ . In fact, this aperture proved too great for observing the light zone located ahead of the missile. On the other hand, a light zone was noticed in the trail that the

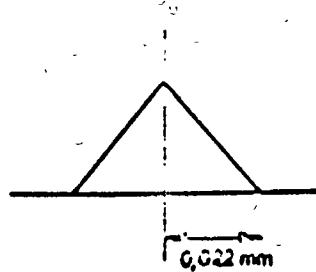


Figure 8.

rotating mirror camera could not spot. To succeed in obtaining both phenomena on the same film, a filter having a density 1 was placed before that portion of the field where the missile nose was during the time of photography. From the exposure standpoint, this set back the aperture to about f 1:3. This figure can be retained to evaluate the minimum blur resulting from adjustment errors in the first set-up used.

In formula (1)

$$G = 0.7$$

$$MP = 5.5 \text{ mm}$$

$$\theta = 1.9$$

hence

$$\delta_{if} = \frac{5.5 \cdot 0.49}{1.9 \cdot 1.7} = 0.8$$

or approximately 1 mm in the space object.

## Diffraction

When rather small relative apertures are used, the diffraction is no longer negligible. In order to compare it to other deficiencies, it is necessary to evaluate its progression. When conventional formulas are used, giving the distance  $x$  of the center of the first 0, in the distribution curve of the light intensity of a "clear" image point, we obtain

$$x = \beta \cdot \lambda \theta (G+1)$$

$\theta$  = relative aperture

$\lambda$  = wave length

$G$  = enlargement between the object and the image.

The value of  $\beta$  varies with the aperture shape (1.22 for a circular aperture) and can be taken as equal to 1, since we have to evaluate a progression.

We can take  $\lambda = 0.5 \cdot 10^{-3}$  mm, which finally will give an approximation of the diffraction spot diameter in the space image

$$2x = \delta_{i,d} = 10^{-3} \theta (G+1) \text{ mm.}$$

If this computation is used for the spark camera, for which we are led to use 42

$$\theta (G+1) = 140,$$

we obtain

$$\delta_{i,d} = 0.14 \text{ mm.}$$

This is low compared to the blur resulting from motion, and it was not possible to bring it out on the densitograms. On the other hand, the contour edge, parallel to the path, is underscored by a rim, having a thickness 0.05-0.1 mm, which can probably be attributed to a diffraction effect.

For the rotating mirror camera, using the above-mentioned values, we obtain

$$\begin{aligned} \delta_{id} &= 0.09 \text{ mm for emitted light} \\ &= 0.11 \text{ mm for shadow setting.} \end{aligned}$$

This is a spread function greater than the setting or velocity defect.

Similarly, for the electronic camera, diffraction is not always negligible.

We will try to find the minimum spread function due to these three defects, whose expressions are listed in the following table.

	Space object	Space image
Motion blur:		
- rotating mirror camera	$\delta_{ov} = \frac{d\alpha}{G}$	$\delta_{iv} = d\alpha$
- other cameras	$\delta_{ov} = \frac{V\theta^2(G+1)^2}{K}$	$\delta_{iv} = \frac{GV\theta^2(G+1)^2}{K}$
Out-of-focus blur	$\delta_{of} = \frac{r}{\theta} \cdot \frac{G}{G+1}$	$\delta_{if} = \frac{r}{\theta} \cdot \frac{G^2}{G+1}$
Diffraction spread function	$\delta_{od} = \frac{10^{-3} \theta (G+1)}{G}$	$\delta_{id} = 10^{-3} \theta (G+1)$

The resultant of several spread functions of an image point is the function obtained from successive convolutions of the composite functions, which, particularly, is not 0 on that interval equal to the sum of the intervals on which the composite function are not 0. An estimation of the total blur is then obtained by saying that it is equal to the sum of the composite blurs; then we have

$$\delta_o = \delta_{ov} + \delta_{of} + \delta_{od}$$

and we will try to minimize it.

According to each case, we have

$$\delta_o = \frac{d\alpha}{G} + \frac{r}{\theta} \frac{G}{G+1} + \frac{10^{-3} \theta (G+1)}{G}$$

$$\delta_o = \frac{V}{K} \theta^2 (G+1)^2 + \frac{r}{\theta} \frac{G}{G+1} + \frac{10^{-3} \theta (G+1)}{G}$$

The minimum is obtained for  $\theta$  and  $G$  tending toward 0 by keeping one in relation to the other in a finite relationship. This is not physically feasible,



because if  $G$  tends toward 0, this means that the picture will be infinitely small.

It is necessary for the dimension of the blurred picture on the film

$$\delta_i = G\delta_o$$

be comparable to the film resolving power.

Let us have

$$\theta(G+1) = u$$

$$\frac{\theta(G+1)}{G} = t$$

$u$  and  $t$  can be chosen arbitrarily, providing they are positive;

$\delta_o$  is then written

$$\delta_o = \frac{d\alpha}{G} + \frac{r}{t} + 10^{-3}t$$

or

$$\delta_o = \frac{V}{K} u^2 + \frac{r}{t} + 10^{-3}t$$

It is the sum of two positive functions with independent variables. The minimum for  $\delta_o$  will be the sum of their minima. The minimum of the  $t$  function is obtained for

$$-\frac{r}{t^2} + 10^{-3} = 0$$

$$\text{i.e., } t_o = \sqrt{1000 r}$$

and the corresponding value is

$$\frac{r}{t_o} + 10^{-3}t_o = 2 \cdot 10^{-3}t_o = \frac{2r}{t_o} = 2 \cdot 10^{-3}t_o.$$

\* The velocity blur minimum shall be obtained for the minimum value of /43  
 $u$  compatible with the physical achievement of the setting.

We see that  $\delta_0$  is limited inferiorly by

$$\delta_{00} = \frac{r}{t_0} + 10^{-3} t_0 = \frac{\sqrt{r}}{5 \sqrt{10}} = 0.063 \sqrt{r}.$$

This seems to be the progression for the best possible resolution for each camera registering a phenomenon having a thickness  $2r$ , in visible light.

The blur resulting from motion is, therefore, minimized independently of the two others. According to the type of camera, we must try to reduce

$$\frac{\alpha}{G}$$

or

$$\frac{V}{K} u^2 = \frac{V}{K} \theta^2 (G+1)^2$$

In order to determine the optimum adjustment of the camera,  $\theta$  and  $G$  must be calculated, taking into account the preceding, and also the fact that the blur dimension set back on the film must not be inferior to the resolving power of the latter.

#### Errors Due to the Film and to the Processing System

The negatives obtained with the cameras are processed with a microdensitometer. We thus obtain a curve in the distance-densities coordinates, from which we should deduct the distance-lighting curve, which has made an impression on the film, to enable reduction of the preceding errors.

An estimation of the film response curve in densities is made by impressing on one end of the film the picture of a tungsten filament seen through filters

having a known density. This can only be an approximate measurement, since it is made with a longer exposure than when photographing the missile, and according to a different spectral composition. We also assume the sensitivity to be uniform throughout the film.

The densitometer slit also has an aperture effect which results in some kind of a glossing of the curve, which can also be beneficial in as much as it reduces the contingent variations of the grain. Analytically, this glossing effect is similar to a blur. Generally speaking, however, we have to use high speed films with a rather large grain, and it is the grain more than the slit that limits the resolution on the densitogram.

The sensitometric curve configuration makes it possible to plot the position of an area edge.

Let us assume that the luminous intensity of the phenomenon varies sharply when crossing the edge (fig. 9a), and also that the spatial punctual response of the observation equipment is symmetric, as represented in figure 9b; the resultant is given in 9c. From this last curve, we can deduce, with a reasonable degree of accuracy, the "edge" position by taking the locus H of the variation. However, if curve 9c is registered on a photographic film, we shall have configuration 9d, on which it will be possible to find point H', image of H, only by calculating the  $\epsilon$  density gap which must exist between the upper level and the H' level, knowing that H is located half-way of the level. If the  $\gamma$  film average is known in the area of H', we show that

$$\epsilon = 0.3\gamma$$

in density units.

In order to use this correction, it is worthwhile to be at sufficiently luminous levels, so as to be far enough away from the bend of the film characteristic curve.

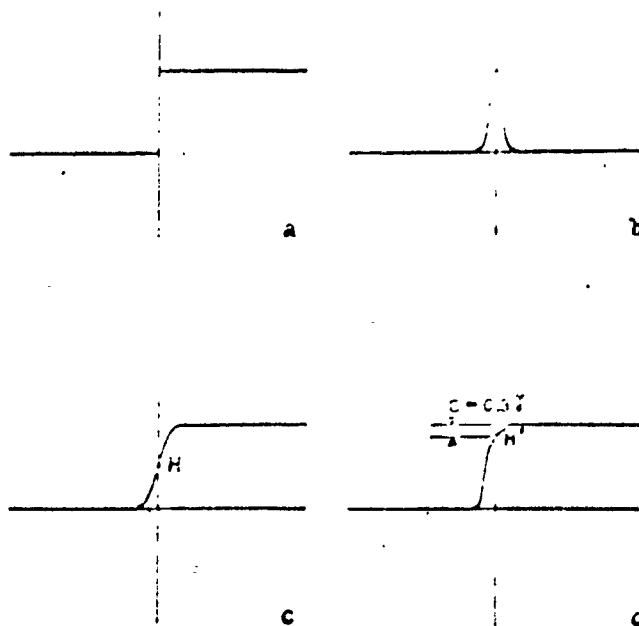


Figure 9

## Conclusion

We had to bring into evidence a minimum blur, set back to the space object, which it does not seem possible to minimize in the case of an object having a  $2r$  thickness, which is that of a cylinder. This value is

$$\delta_{00} = 0.063 \sqrt{r} \quad r \text{ and } \delta_{00} \text{ in mm.}$$

The blur resulting from the missile motion is cumulated. The following remarks can be made:

The 0.063 coefficient is no doubt somewhat pessimistic. Indeed, the blur does not have the shape of a square crenel having a  $\delta_{00}$  width, but rather the configuration given in figure 10. In a number of applications, it will be possible to determine an effective width ranging from  $\delta_{00}$  and  $\delta_{00}/2$ .

For a given form, it is better to work with the greatest possible caliber. Indeed, the relative error will diminish when  $r$  increases:

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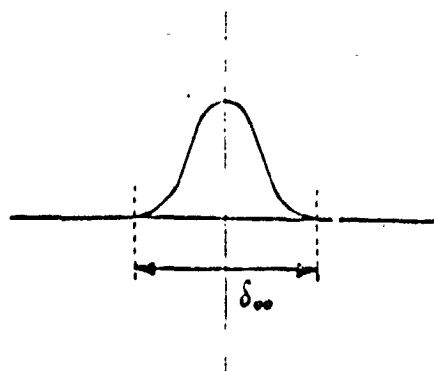


Figure 10

$$\frac{\delta_{\infty}}{r} = \frac{0.063}{r}$$

The velocity compensation must always be worthwhile when just enough light is available. In fact, the enlargement and aperture settings are made in such a way as to reduce the other sources of blur. The velocity compensation makes it possible to have a sufficient exposure, without having a significant blur.

Finally, it seems interesting to develop methods which make it possible to obtain an effective resolution better than the blur dimension, often considered as a limit of the resolving power.

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